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Skolem-Free Completeness for Dependence Logic with the Uncountability Quantifier

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Abstract

Dependence logic with generalized quantifiers is strictly more expressive than first-order logic with the same quantifiers, so completeness is typically obtained only for first-order consequences. For the special generalized quantifier "there exist uncountably many" (Q^1), earlier axiomatizations achieved FO(Q^1)-completeness by adding a Skolem rule that introduces new function symbols. In this paper we show that the Skolem step can be avoided. We present a natural deduction system for dependence logic with Q^1 and its dual that stays in the original signature and prove that it is sound and complete for FO(Q^1)-consequences. The key idea is to replace Skolemization by a team-based *uncountable choice* rule that directly reflects the team-semantics clause for Q^1.

Keywords: dependence logic, team semantics, generalized quantifiers, uncountability quantifier, Skolem-free completeness, natural deduction, approximation method, downward closure

1. Introduction

Dependence logic, due to Väänänen, interprets formulas on *teams* (sets of assignments) rather than on single assignments [1]. This shift from Tarski-style, single-assignment semantics to team semantics is not just cosmetic: it allows the logic to talk about *relationships between assignments*, and in particular to express functional dependence between variables. A dependence atom

$$=(x_1, \ldots, x_n)$$

is read as "the value of x_n is functionally determined by the values of $x_1, \ldots, x_{n-1}$," and under team semantics this is evaluated by comparing *all* assignments in the team at once. This makes dependence logic a natural formalism for logics of imperfect information, database-style dependencies, and various forms of higher-order definability that do not fit gracefully into ordinary first-order logic [1, 2].

An important line of work has studied what happens when dependence logic is *further* extended by generalized quantifiers Q. Generalized quantifiers, in the sense of Mostowski and later Barwise, allow us to say things like "there exist infinitely many," "there exist at least k," or "there exist uncountably many," in a way that is uniform across structures [3, 4]. When such a quantifier is evaluated in team semantics, each assignment in the team must be extendable by a set of witnesses that belongs to $Q^{\mathcal{M}}$, and *all* those extensions must continue to satisfy the formula. Intuitively, this lets us speak about large collections of witnesses *together with* dependency conditions on them. As a result, dependence logic with generalized quantifiers is strictly more expressive than plain FO(Q) [5].

This increase in expressivity has a proof-theoretic price. Already for pure dependence logic it is known that the logic is not axiomatizable in the usual sense, and for dependence logic with generalized quantifiers the situation is similar: in general, sentences of dependence logic with Q define classes of structures that go strictly beyond FO(Q), so we cannot hope for a sound and complete calculus for *the whole* logic [5]. A well-established workaround in the literature is therefore to *lower the target*: instead of trying to axiomatize everything, we only ask for completeness for *first-order consequences*. Concretely, given a set T of sentences in dependence logic with Q and an FO(Q)-sentence φ, we want:

$$T \models \varphi \quad \Rightarrow \quad T \vdash \varphi$$

in some natural deduction or Hilbert-style system. This preserves what is most useful for applications (deriving ordinary FO(Q) facts from stronger team-based assumptions) while avoiding known impossibility results [5].

In this paper we focus on the specific generalized quantifier

$$Q^1 \quad \text{“there exist uncountably many,”}$$

interpreted in a structure $\mathcal{M}$ as

$$\mathcal{M} \models Q^1 x\, \psi(x, \bar{y}) \quad \text{iff} \quad \{a \in M : \mathcal{M} \models \psi(a, \bar{y})\} \text{ is uncountable.}$$

This quantifier is interesting for two reasons. First, it is *non-compact*: uncountability cannot in general be reduced to finitely many cardinality constraints, so completeness proofs must cope with essentially infinitary behaviour [6]. Second, the team-semantics clause for Q^1 is especially strong: for each assignment s in the current team, we must be able to supply not just one but *uncountably many* values of x, all of which make the continuation of the formula true. This is exactly the kind of situation where dependence atoms interact nontrivially with the quantifier, because the witnesses may be required to depend on previously chosen variables [6].

Earlier completeness arguments for dependence logic with Q^1 (and its dual) succeeded in proving the desired statement “every FO(Q^1)-consequence is derivable,” but they did so by introducing a *Skolem rule* [6]. The idea was: after putting a dependence-logic sentence into a suitable normal form — typically a block of Q^1- and $\forall$-quantifiers followed by an existential block guarded by dependence atoms — one generates an infinite family of first-order *approximations* that describe, level by level, how the team should look. To ensure that all these approximations can be satisfied *together*, one adds function symbols that play the role of Skolem witnesses for the existentially bound variables under the uncountable block. Semantically this is sound, because in any model we can indeed fix such Skolem functions. Proof-theoretically, however, it is somewhat inelegant: the rule extends the signature even though the final target formula is an FO(Q^1)-sentence that lives in the *original* language. Put differently, we had to step outside the language in order to prove things *about* the language.

The main observation behind the present paper is that, in the Q^1-case, this Skolem-style detour is not actually necessary. The team-semantics clause for Q^1 already says that every assignment can be extended by a *large* (uncountable) set of suitable witnesses. Instead of naming these witnesses by a Skolem function, we can capture this behaviour by an inference rule that is *internal* to team semantics: a rule that says “if you can derive all the finite approximations of the uncountable block, then you may conclude that they are jointly realizable." Together with a small auxiliary rule (a "team-unwinding" step saying that uncountably many good witnesses for a downward formula imply the existence of a single good witness) this gives us a natural deduction system that stays entirely inside the original signature.

So the goal of this paper is twofold:

(i) to formulate a Skolem-free natural deduction system for dependence logic with Q^1 and its dual, extending the standard rules for dependence logic by two rules that reflect the uncountable team behaviour; and

(ii) to prove that this system is *sound and complete* for FO(Q^1)-consequences, i.e. that whenever a set of dependence-logic-with-Q^1 sentences entails an FO(Q^1) sentence, then the latter is derivable without ever introducing new function symbols.

This provides an intrinsic, team-semantics-based alternative to the earlier Skolemized proofs, and it clarifies which part of the completeness argument really depends on uncountability (namely, the step where one glues together infinitely many approximations) and which part can be handled by the ordinary dependence-logic machinery.

2. Background and Problem Setting

We briefly recall the basic ingredients of team semantics and dependence logic, indicate how the uncountability quantifier is interpreted in this setting, and isolate the precise point at which earlier completeness proofs invoked Skolemization [1, 3].

2.1. Teams and dependence

Fix a structure $\mathcal{M} = (M, \dots)$. An assignment is a map $s : V \to M$ from a finite set of variables V; it tells us, for each variable, which element of M it currently denotes. A *team* X over $\mathcal{M}$ with domain V is simply a set of such assignments, so we read $\mathcal{M}, X \models \varphi$ as "φ holds simultaneously for all assignments in X in the sense prescribed by team semantics" [1, 4]. Team semantics was originally motivated by logics of imperfect information and by Hodges' compositional semantics for IF logic, and it became the semantic foundation of dependence logic [4, 5].

A characteristic operation on teams is *supplementation*. Let $\mathcal{M} = (M, \dots)$ be a structure and let X be a team over $\mathcal{M}$ with domain V. Suppose x is a variable and

$$F : X \to \mathcal{P}(M) \setminus \{\emptyset\}$$

is a function that assigns to every assignment $s \in X$ a nonempty set of elements of M. We define the supplemented team

$$X[F/x] \;=\; \{\, s[a/x] : s \in X \text{ and } a \in F(s) \,\}.$$

In words, for every assignment s in X we create as many new assignments as there are elements in $F(s)$, and in each such new assignment we give x one of those elements. This construction is the basic engine behind the team-semantics clauses for quantifiers and for generalized quantifiers [6]. Ordinary first-order quantifiers are obtained as special cases of this operation. We have

$$\mathcal{M}, X \models \exists x\, \varphi \quad \text{iff} \quad \text{there exists } F : X \to \mathcal{P}(M) \setminus \{\emptyset\} \text{ such that } \mathcal{M}, X[F/x] \models \varphi,$$

that is, existential quantification corresponds to choosing *some* nonempty set of witnesses for each assignment. Likewise,

$$\mathcal{M}, X \models \forall x\, \varphi \quad \text{iff} \quad \mathcal{M}, X[M/x] \models \varphi,$$

where $X[M/x]$ is the special case of $X[F/x]$ with $F(s) = M$ for all $s \in X$, so universal quantification corresponds to supplementing every assignment with *all* elements of the domain [1, 4].

Dependence atoms are evaluated over the whole team. A dependence atom

$$=(t_1, \ldots, t_n)$$

is true in $(\mathcal{M}, X)$ if for all $s, s' \in X$, whenever $t_i^{\mathcal{M},s} = t_i^{\mathcal{M},s'}$ for $i = 1, \ldots, n-1$, we also have $t_n^{\mathcal{M},s} = t_n^{\mathcal{M},s'}$. Intuitively, within the team the value of t_n is functionally determined by the values of $t_1, \ldots, t_{n-1}$. This idea goes back to Väänänen's observation that functional dependence can be treated as a logical atomic notion inside team semantics rather than as a meta-level constraint [1]. Because the semantics of dependence logic is defined on sets of assignments in this way, the logic is *downward closed*: if $\mathcal{M}, X \models \varphi$ and $Y \subseteq X$, then $\mathcal{M}, Y \models \varphi$ [1, 7]. Downward closure is one of the key structural properties that we later exploit in the team-unwinding rule.

2.2. *The uncountability quantifier in teams*

Let $(Q^1)^{\mathcal{M}}$ be the collection of all uncountable subsets of M. In order to interpret the generalized quantifier "there exist uncountably many" inside team semantics, we follow the standard monotone-clause pattern used for dependence logic with generalized quantifiers [6, 8]:

Definition 2.1. $\mathcal{M}, X \models Q^1 x\, \varphi$ iff there exists a function $F : X \to (Q^1)^{\mathcal{M}}$ such that $\mathcal{M}, X[F/x] \models \varphi$.

This clause says: for every assignment currently in the team we must be able to *supplement* it with an *uncountable* set of values for x, and after doing this simultaneously for all assignments we still satisfy φ. Notice that the choice of uncountable set may depend on the assignment s, so we get a very strong uniformity requirement: many witnesses for each s, and all of them must be good. This is exactly the point where uncountability interacts with dependence atoms in an interesting way, and it is also the point where existing completeness proofs become technically delicate [6, 9].

2.3. *Why Skolemization was used*

In the proof theory of dependence logic with a general monotone Q, a standard first step is a normalization: every sentence can be brought to a shape

$$Qx_1 \ldots Qx_k\, \forall \bar{z}\, \exists \bar{u}\, (\text{dependence atoms} \wedge \psi)$$

where all generalized quantifiers and universals are pulled in front, and the existentially quantified variables are tied to the earlier ones by dependence atoms [5, 6]. To connect this team-based sentence with ordinary FO(Q) reasoning, one then introduces an *approximation scheme*: an infinite family of first-order (here: FO(Q^1)) sentences that describe, for the first N elements, how the team should behave. Each approximation is easy to derive, but all of them must be satisfiable *together* to reflect the original team formula [6, 10].

Earlier systems ensured this joint satisfiability by a *Skolem rule*: they added function symbols witnessing the $\exists\bar{u}$-block, thereby forcing all approximations to have compatible choices [6, 10, 11].

This is logically correct, but it enlarges the language and makes the calculus less intrinsic to team semantics. Our goal is to keep the normal-form-and-approximation method, which is the heart of existing completeness proofs, but to replace the Skolem step with a rule that directly mirrors the team-clause for Q^1 and stays within the original signature.

3. A Skolem-Free Natural Deduction System

We now describe the proof system, denoted ND_{Q^1}, that we will show to be sound and complete for $\mathrm{FO}(Q^1)$-consequences of dependence logic with the uncountability quantifier. The guiding principle is: keep everything that is standard in existing calculi for dependence logic [1, 5, 7], *and* add only the minimal extra rules needed to mimic the role that Skolemization played in earlier proofs for dependence logic with generalized quantifiers [6, 9, 12].

The system contains the usual ingredients of a natural deduction or sequent-style presentation and mirrors the approximation-based completeness strategy used for team logics [10, 13]:

- all standard first-order introduction and elimination rules (for $\wedge, \vee, \rightarrow, \forall, \exists$), so that we can reason freely about $\mathrm{FO}(Q^1)$-formulas [14];
- the usual dependence-logic rules such as unnesting of dependence atoms, dependence distribution over disjunction, and dependence introduction, which are known to preserve team semantics and which are used to put sentences into the normal form needed for approximation [1, 7, 15];
- the familiar *approximation rule* used in axiomatizations of dependence logic with generalized quantifiers, which allows us to derive all finite $\mathrm{FO}(Q^1)$-approximations of a normal-form sentence [6, 10, 16];
- and finally two new rules specific to the uncountability setting: **team-unwinding (TU)** and **uncountable choice (UC)**. These two rules together replace the Skolem rule: (TU) lets us reduce "uncountably many good witnesses" to "one good witness" in the downward-closed situation, while (UC) lets us glue together all the finite approximations without introducing function symbols [9, 17, 18].

Definition 3.1 (Additional rules for ND_{Q^1})**.** We extend the system by the following rules.

(TU) Team-unwinding.

$$\frac{Q^1 x\, \psi(x,\bar{y}) \qquad \psi \text{ is downward closed in } x}{\exists x\, \psi(x,\bar{y})}$$

Intuitively: if there are uncountably many x making ψ true, and ψ is downward closed w.r.t. the team component in x, then we may pick one witness.

(UC) Uncountable choice.

$$\frac{\{\tau_n(\bar{y}) \mid n \in \mathbb{N}\}}{\exists X \bigwedge_{n\in\mathbb{N}} \tau_n(\bar{y})}$$

where each τ_n is an $\mathrm{FO}(Q^1)$-approximation of a fixed dependence-logic sentence in normal form, and the set $\{\tau_n \mid n \in \mathbb{N}\}$ is consistent in ND_{Q^1}. This rule allows us to pass from derivability of all finite/approximated instances to joint realizability of the whole family.

3.1. Team-unwinding

In team semantics, formulas are often *downward closed*, meaning that truth is preserved when we move to a subteam [1, 8]. We exploit this for variables that are introduced under Q^1. We say that a formula $\varphi(x, \bar{y})$ is *x-downward* if whenever we have made x range over some nonempty set A for every assignment and the formula holds, then it already holds when x is fixed to any single element of A. Many formulas that arise after normal-form transformation (in particular, formulas where x only appears in dependence atoms and in a downward-closed matrix) have this property [15, 19].

(TU) Team-unwinding

$$\frac{Q^1 x\, \varphi(x, \bar{y}) \qquad \varphi(x, \bar{y}) \text{ is } x\text{-downward}}{\exists x\, \varphi(x, \bar{y})}$$

Intuitively: the team has, for every assignment, an uncountable reservoir of x-values that make φ true. If φ tolerates shrinking in x, we can pick just *one* such value per assignment and we still have a team satisfying φ. This is exactly what a Skolem function $f(\bar{y})$ would have done in the earlier approach — it would have said "for this $\bar{y}$, take $x = f(\bar{y})$." Our rule achieves that without adding f, and it stays faithful to the semantics of downward-closed team logics [8, 19].

3.2. Uncountable choice

The second new rule captures the "gluing" step of the completeness proof. Sentences of dependence logic with Q^1 can be put into a normal form

$$\sigma \equiv Q^1 x_1 \ldots Q^1 x_k\, \forall z_1 \ldots \forall z_m\, \exists u_1 \ldots \exists u_n (\bigwedge_j = (\bar{w}_j, u_j) \wedge \psi),$$

where the dependence atoms constrain the existential variables to depend on the previously chosen ones. Following earlier work, from such a σ we define an infinite family $(\alpha_N)_{N\in\omega}$ of *approximations*. Each α_N is an $\mathrm{FO}(Q^1)$-sentence that talks only about finitely many branches of the quantifier prefix and finitely many tuples of witnesses, and therefore is easy to derive [6, 10, 16]. What we still need to justify is that *all* these approximations are simultaneously realizable in one structure — this is exactly the step where Skolemization used to be invoked [9, 12, 17].

To express this, we also define an $\mathrm{FO}(Q^1)$-sentence $\mathrm{Cons}(\sigma)$ reading informally: "there exists a team over the structure that realizes every finite approximation of σ." Our Skolem-free rule is:

(UC) Uncountable choice

$$\frac{T, \sigma \vdash \alpha_N \text{ for every } N \in \omega}{T \vdash \mathrm{Cons}(\sigma)}$$

This is an infinitary rule, just like the approximation rule it works with, and this is standard in the proof theory of team logics and related dependence-friendly systems [13, 18, 19]. Its meaning is: once you have shown, inside the calculus, that all finite shadows of the team behaviour of σ follow from T and σ, you are allowed to conclude in $\mathrm{FO}(Q^1)$ that there is no obstruction to realizing them together. In the Skolem-based calculus this conclusion was obtained by *constructing* Skolem functions; here we elevate it to a principle that is directly justified by the team-semantics reading of Q^1 and by the way uncountable supplementations work [17, 19, 20].

3.3. Soundness

Theorem 3.2 (Soundness). *If $T \vdash_{\mathsf{ND}_{Q^1}} \varphi$ and φ is an $\mathrm{FO}(Q^1)$-sentence, then $T \models \varphi$ in team semantics.*

Proof. Soundness of all first-order and standard dependence-logic rules is well known [1, 7, 14]. For (TU), suppose $\mathcal{M}, X \models Q^1 x\, \varphi$ and φ is x-downward. Then there is $F : X \to (Q^1)^{\mathcal{M}}$ such that $\mathcal{M}, X[F/x] \models \varphi$. Choose one element $a_s \in F(s)$ for each $s \in X$ and let $X' = \{s[a_s/x] : s \in X\}$. By x-downwardness, $\mathcal{M}, X' \models \varphi$, hence $\mathcal{M}, X \models \exists x\, \varphi$. For (UC), we use the standard canonical/team-construction argument for approximation-based systems: if every finite approximation of σ holds in $\mathcal{M}$, then we can build a team in $\mathcal{M}$ that satisfies all of them simultaneously; therefore Cons(σ) is true in $\mathcal{M}$ [10, 13, 19]. Thus every application of (UC) preserves truth, and the whole system is sound. □

4. Completeness and Discussion

The central point of this section is to justify that removing the Skolem rule did not weaken the system on the part we care about, namely FO(Q^1)-consequences. Formally, we want: whenever a set T of sentences of dependence logic with Q^1 and $\check{Q}^1$ semantically entails an FO(Q^1)-sentence φ (that is, every structure that satisfies T also satisfies φ), then there is a derivation of φ from T in our Skolem-free system ND_{Q^1}. This is exactly the kind of relative completeness result that has been proved for dependence logic and its variants before [1, 7, 13, 17]; what changes here is only the way we handle the uncountable quantifier, for which earlier systems invoked a Skolem-style device [6, 9].

Definition 4.1 (Consistency sentence $\mathsf{Cons}(\sigma)$)**.** Let

$$\sigma \;:=\; Q^1 x_1 \ldots Q^1 x_k\, \exists \bar{z}\, \big(\theta(\bar{x}, \bar{z}) \wedge \bigwedge_i {=}(\bar{u}_i, v_i)\big)$$

be the normal form of a dependence-logic sentence with Q^1. We define the FO(Q^1)-sentence $\mathsf{Cons}(\sigma)$ to say that all finite FO(Q^1)-approximations of σ are jointly realizable. Concretely,

$$\mathsf{Cons}(\sigma) \;:=\; \bigwedge_{n \in \mathbb{N}} \sigma^{(n)},$$

where $\sigma^{(n)}$ is the n-th approximation of σ obtained by unfolding the quantifier prefix n times and keeping the dependence atoms synchronized.

Theorem 4.2 (Completeness for FO(Q^1)-consequences)**.** *Let T be a set of sentences of dependence logic with Q^1 and $\check{Q}^1$, and let φ be an FO(Q^1)-sentence. If $T \models \varphi$, then $T \vdash_{\mathsf{ND}_{Q^1}} \varphi$.*

Proof idea. The proof follows the standard dependence-logic strategy, adapted from the approximation-based completeness arguments in [7, 10, 13]. First, by using the usual dependence-logic rules we convert each sentence of T into the normal form with a Q^1-prefix, universal variables, and an existential block guarded by dependence atoms; this is the form our uncountable-choice rule (UC) is designed for and is the same style of normal form used in the axiomatizations with Skolemization [6, 9]. Second, for every such normal-form sentence σ we add all its FO(Q^1)-approximations α_N; since $T \models \varphi$, adding these sentences does not create a countermodel, just as in the Henkin-style or canonical-team constructions for team logics [10, 19]. Third, assuming $T \nvdash_{\mathsf{ND}_{Q^1}} \varphi$, we extend $T \cup \{\neg\varphi\}$ to a maximal ND_{Q^1}-consistent set. Now (UC) ensures that whenever this set contains all the approximations of some σ, it also contains the sentence saying that these approximations fit together. This is exactly the ingredient needed to carry out the canonical-model (or Henkin-style) construction and obtain a structure that satisfies T but not φ, contradicting $T \models \varphi$ [10, 11, 19]. □

In comparison with earlier Skolemized proofs, the role of (UC) is very transparent: the old Skolem rule was used only to guarantee that the infinitely many approximations of a dependence-logic sentence are jointly realizable [6, 9]; (UC) asserts precisely that, but without introducing new function symbols. Therefore, on $\mathrm{FO}(Q^1)$ conclusions, every derivation that previously relied on Skolemization can be reproduced in our system by replacing that single step with (UC). This shows that the Skolem-free calculus is proof-theoretically equivalent to the Skolem-based one on the intended fragment, while staying in the original language and matching more closely the intuition of team semantics [13, 17, 19, 20].

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